

PHYSICS OF PARTICLE DETECTION

- We detect particles by exploiting some sort of interaction of the particles with the material in the detector.

⇒ We need to understand interaction of particles with matter

- There are "short-range" strong and weak interactions, which at high energies can produce new particles

- There are "long-range" electromagnetic interactions
 - not widely used; mostly non-destructive

- Charged particles passing through matter

- lose energy

→ ionize atoms in the material due to interactions with atomic electrons (ionization energy loss)

→ produce electromagnetic radiation - Čerenkov, transition radiation, scintillation, etc. (radiative energy loss)

- scatter

→ primarily due to interactions with atomic nuclei

- Photons are detected/measured via photoelectric effect, Compton scattering or pair production

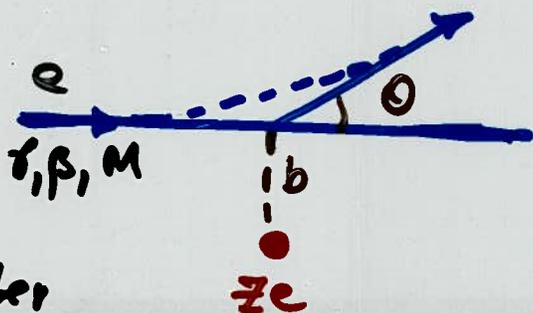
- Neutrons and neutrinos are detected via strong and weak interactions, respectively.

Single Scattering

Coulomb collision of a

charged particle with charge "e", mass m, velocity βc

by a nucleus with atomic number "Ze" at an impact parameter b.



→ Scatters off at an angle θ

Coulomb force $\sim 1/r^2$ and active on time scales where the incident particle is near the scattering center.

$$F(b) = Ze \cdot e / b^2$$

$$\Delta t = 2b/v$$

Net momentum impulse,

$$\Delta p_T \sim F(b) \cdot \Delta t = 2Ze^2/bv$$

← characteristic time

$$\text{Scattering angle, } \theta \sim \frac{\Delta p_T}{p} = \frac{2Ze^2}{pbv} \sim \frac{2Z\alpha}{pbv}$$

$$\theta_R \sim \frac{Z\alpha}{b} \cdot \frac{1}{T} = \frac{U(b)}{T} \leftarrow \text{P.E. / K.E.} \quad (2)$$

Minimum and Maximum Scattering Angles

The extent of electron cloud $\sim A$

If the incident particle is antihelium \sim Bohr radius, the Coulomb field of the nucleus is screened by the electrons.

\Rightarrow A minimum scattering angle

$$\theta_{\min} \sim \frac{2Ze}{p v \cdot a_0}$$

(Saves the cross section from diverging despite $1/\theta^4$)

Also, for particle wavelength $\lambda \sim R$, the scattering angle is confined to be smaller than

$$\theta_{\max} \approx \frac{\lambda}{R} = \frac{h}{pR} \approx \frac{2 \cdot mc}{p \alpha A^{1/3}}$$

$$R \approx r_0 A^{1/3}$$
$$r_0 \approx 1 \text{ fm}$$

The total elastic scattering cross section then is estimated by

$$\sigma = \int_{\theta_{\min}}^{\theta_{\max}} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta \, d\phi$$

Scattering Cross Section

For impact parameter between b and $b+db$,

$$d\sigma = b \cdot db \cdot d\phi$$

$$d\sigma = \frac{d\sigma}{d\Omega} \cdot d\Omega = \frac{d\sigma}{d\Omega} \cdot \sin\theta \cdot d\theta \cdot d\phi$$

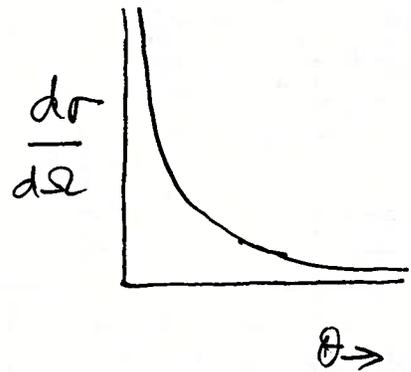
$$d\phi = 2\pi$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \cdot \left(\frac{db}{d\theta} \right)$$

Differential cross section

$$\theta \sim \frac{2Z\alpha}{p v b}, \text{ as shown previously.}$$

$$\frac{d\sigma}{d\Omega} \sim \left(\frac{2Z\alpha}{m v^2} \right)^2 \cdot \frac{1}{\theta^4}$$



Recall Rutherford formula

$$\frac{d\sigma_R}{d\Omega} \propto \frac{1}{\sin^4 \theta/2}$$

valid for non-relativistic spin-0 particles.

For spin $\frac{1}{2}$ particles,

$$\text{Mott Formula} \quad \frac{d\sigma_M}{d\Omega} = \frac{d\sigma_R}{d\Omega} (1 - \beta^2 \sin^2 \theta/2)$$

Reduces to Rutherford form for $\beta \rightarrow 0$, $\theta \rightarrow 0$.

Multiple Coulomb Scattering

Total cross section $\sim \frac{1}{\theta^4}$

$$\sigma = \int_{\theta_{\min}}^{\theta_{\max}} \frac{d\sigma}{d\Omega} 2\pi\theta \cdot d\theta = \int_0^{a_0} 2\pi b db$$

Geometric cross section $\Rightarrow \pi a_0^2 \sim \frac{1}{\theta_{\min}^2}$

Mean ^{scattering} angle is the angle weighted by the $\ln \theta$ angular distribution

$$\langle \theta^2 \rangle = \frac{\int \theta^2 \frac{d\sigma}{d\Omega} \cdot d\Omega}{\int \frac{d\sigma}{d\Omega} \cdot d\Omega} = \frac{\int d\theta / \theta}{\int \frac{d\theta}{\theta^3}}$$

$$\sim 2 \theta_{\min}^2 \left[\ln \frac{\theta_{\max}}{\theta_{\min}} \right]$$

When the particles traverse the medium, let us say there are N scatters along the way, on average

$$\langle \theta_{MS}^2 \rangle = N \langle \theta^2 \rangle$$

$$N = \frac{N_0 \rho \sigma}{A} \cdot dx = \frac{dx}{\langle L \rangle}$$

Mean free path between scatters

$$\langle \theta_{MS}^2 \rangle = \frac{N_0 \rho dx}{A} \cdot 2\pi \left[\frac{2Z\alpha}{p\beta c} \right]^2 \ln(\dots)$$

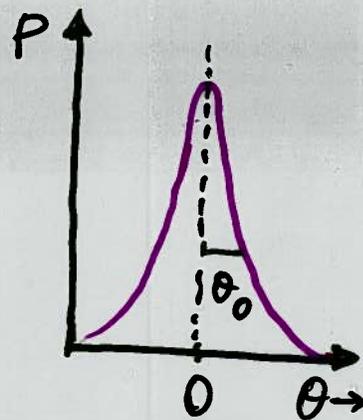
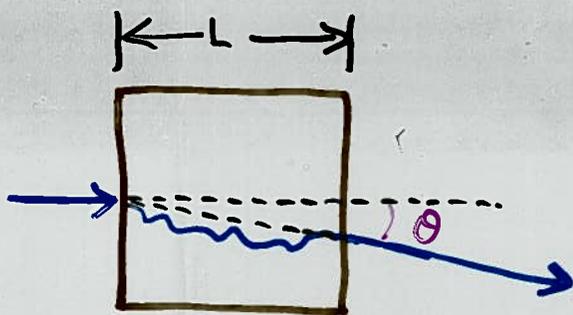
$$\theta_0 = \sqrt{\langle \theta_{MS}^2 \rangle} = \frac{E_S}{p\beta} \sqrt{\frac{dx}{x_0}}$$

$$E_S = \sqrt{\frac{4\pi}{\alpha}} (mc^2) = 21 \text{ MeV}$$

A 20 GeV pion, for example, in one radiation length, suffers a mean scattering by angle $\theta \sim 0.001$ rad or 1 mrad

Multiple Coulomb Scattering

- A charged particle traversing a medium can suffer many small angle scatters in the Coulomb potentials of nuclei

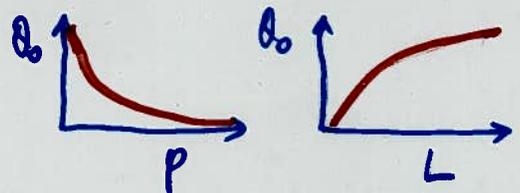


$$P(\theta) = \frac{1}{\sqrt{2\pi}\theta_0} \exp\left(-\frac{\theta^2}{2\theta_0^2}\right)$$

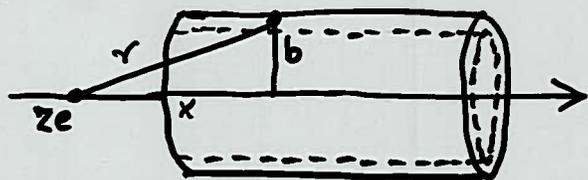
$$\theta_0 = \sqrt{\langle \theta^2 \rangle} = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{\frac{L}{X_0}} \left[1 + 0.038 \ln \frac{L}{X_0} \right]$$

p : momentum in MeV/c, βc : velocity, z : charge of the scattered particle, X_0 : Radiation length

$$\theta_0 \approx \frac{13.6}{\beta c p} \sqrt{\frac{L}{X_0}}$$



Energy Loss by Ionization



consider a heavy charged particle with charge ze and velocity v passing through some material.

The momentum transferred to an electron in the material, at a distance r and impact parameter b is,

$$\Delta p = \int_0^{\infty} F_{\perp} dt = \int_{-\infty}^{+\infty} \frac{ze \cdot e}{r^2} \cdot \frac{b}{r} \cdot \frac{dx}{v}$$

$$= \frac{ze^2}{v} \int_{-\infty}^{+\infty} \frac{b \cdot dx}{(\sqrt{b^2 + x^2})^3} = \frac{ze^2}{v \cdot b} \int_{-\infty}^{+\infty} \frac{d(x/b)}{\underbrace{\left(\sqrt{1 + (x/b)^2}\right)^3}_{= 2}}$$

$$= \frac{2ze^2}{b \cdot v} \quad \text{--- (1)}$$

\therefore The energy transfer is,

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{(2ze^2/bv)^2}{2m} = \frac{2ze^4}{mb^2v^2} \quad \text{--- (2)}$$

(8)